new methods of medical treatment for infected man. (2) Systematic instruction and tactful control of the peoples affected. This will be the duty of the medical and teaching professions of the stricken countries. Anyone who has watched the increase of well-taught and capable physicians in such a country as India during the last twenty years will base great hopes on the growth of this influence. (3) And most important, a common and indignant consciousness that these plagues are not inevitable, that by combined effort they can be cast off, and that it is a disgrace to humanity that one-half of its members should be harbouring these loathsome parasites.

The Theory of Relativity in Relation to Scientific Method.[1]

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SOME interesting criticisms of the theory of relativity have been advanced recently by M. Paul Painlevé, in two papers in the *Comptes rendus de l'Académie des Sciences de Paris*.[2] M. Painlevé attacks the theory as it at present stands, on grounds which are of general scientific interest. He criticises the expression for ds, the element of length adopted by Einstein,

$$ds^2 = dt^2(1 - a/r) - r^2(d\theta^2 + \sin^2\theta\, d\phi^2) - dr^2/(1 - a/r)$$

on the ground that it is one of a very large number of forms which satisfy the Einstein conditions. He cites some of the other possible forms for the relation between the length element and the four co-ordinates (r, θ, ϕ, t), and indicates the various consequences which ensue according to the particular form adopted.

At this point we encounter, as M. Painlevé points out, a serious difficulty; but it is a difficulty which is present in all scientific investigations. The botanist plotting on paper the results of experiments which were designed to discover the relation between two variables, x and y, is faced by the same problem when he decides on the method to be adopted in interpolation. For his experiments merely tell him that, whatever the relation between the variables may be, the function connecting them must be such that when $x = x_r$, we also have $y = y_r$, where (x_1, y_1), (x_2, y_2) . . . (x_r, y_r) . . . (x_n, y_n) represent, roughly speaking, the results of his experiments. But the number of his observations is necessarily finite; and it is evident that there are at least as many functions satisfying these conditions as there are points in the mathematical continuum. This difficulty of choosing between a set of functions all of which satisfy the data of the problem presents itself at several critical points of the Einstein theory. It is entirely plain that if science is to be possible, some further principle is required.

THE SIMPLICITY POSTULATE.

In the face of this difficulty, it has been the practice of scientific writers to choose the simplest function available. The question of what constitutes simplicity, or rather the question of when one function is simpler than another, is a difficult one, but in ordinary scientific work, and especially in biology, the term is considered to be well understood. In selecting the simplest alternative, no one, of course, would hold that the other alternatives are impossible. Indeed, the simplicity criterion arranges the various possibilities in serial order. If the first of this set afterwards proves unsuitable, the next one is taken, and so on. Thus, in outline, we may say that the procedure of science is to attach probabilities to the various functions in such a way that the probabilities of functions arranged in order of simplicity decrease rapidly to zero, so that there is little probability of any of the more complicated functions which could be devised being the correct one.

In criticising this procedure from a logical point of view, it will be of no avail to demand, at the outset, a definition of the relation involved in the proposition that one function *is simpler than* another. Common sense uses the notion of simplicity, and we cannot go behind common sense. The business of the logician is to interpret it and relate its various beliefs *inter se*, eliminating when necessary the less fundamental beliefs in favour of those which are held more firmly and the deductions which can be drawn from these beliefs. But this absence of definition makes it important to consider the way in which the simplicity postulate is used in relativity theory. M. Painlevé discusses some of the alternative forms for the length element, to which he sees no objection. He shows that some of them carry with them consequences as to the change in dimensions of a moving body which are mutually inconsistent and in direct contradiction to the Einstein theory. It may therefore be possible to make a choice between some of them by means of data of this kind, and consequently to settle the controversy as to the form of ds, at least to the extent of eliminating those forms which give certain types of change in the dimensions of bodies in motion. M. Painlevé states that he considers some of his forms to be as simple as the form adopted by Einstein. In the absence of a decision being reached by means of further data, the objection of M. Painlevé will fall to the ground only if it is established that the form which Einstein has used for the length element is the simplest one which fits the facts of the external world.

THE VALUE OF COMPREHENSIVENESS.

There is another logical property which enables us to assign a value to rival scientific theories. In choosing between various ways of relating facts *inter se*, we shall evidently prefer theories which group together the largest number of facts under one set of assumptions. Comprehensiveness is, indeed, an important test of the value of a theory, for as the number of facts which are linked together by a theory increases, the theory grows in importance as a

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[1] Paper read before the Congress of Philosophy in Paris on December 29, 1921.

[2] "La Mécanique classique et la théorie de la relativité," October 24, 1921; "La Gravitation dans la mécanique de Newton et dans la mécanique d'Einstein," November 14, 1921.

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hypothesis, and is of greater value as a guide to the selection of future researches. The recent developments of the theory of relativity due to Profs. Weyl and Eddington are of considerable importance as examples of the value of increasing the range of a theory. Weyl has generalised the geometry used by Einstein in order to produce a function which can conveniently be made to represent the electro-magnetic energy tensor; and Eddington, in accordance with the methodological considerations mentioned above, has suggested still more radical generalisations, with the view of producing, if possible, some function which can be used as an electronic energy tensor. By this we mean a function which contains at least analogues of the main properties of the electron. Towards this very important result Eddington has taken several significant steps, though the physical aspect of this part of the energy of a system, associated with the non-Maxwellian forces, is by no means clear at present. It will obviously be a matter of the greatest importance if it proves possible to cover the electronic phenomena as well as the gravitational and electro-magnetic by a few perfectly definite general assumptions of the same type as those already introduced in relativity theory.

Among the results obtained by Eddington, we may direct attention to the fact that a natural unit of action has made its appearance in terms of which both the energy tensor and the electro-magnetic tensor can be expressed. It appears that this unit of action is 10^{114} times the quantum required in the quantum theory, but the fact that the two energy tensors, which so far have been treated on the lines of world geometry, can be given in terms of the one unit of action may well suggest further developments which may accomplish ultimately the introduction of a tensor to represent the electronic or non-Maxwellian forces.

But let us consider how these advances have been brought about. On Weyl's theory, it is possible that comparisons of length at different times and at different places may yield discordant results according to the route of comparison. In fact, a particular standard of length should apparently be used only at the time and place where it is, for in general, a vector will change its value on describing a circuit. The fundamental apparatus required for measurement is therefore no longer, as in the days before relativity, a unit standard, or indeed, a set of standards, one for each point of space, but a set containing a unit for each point of the fourfold manifold of space and time. Such a system of measures, comprising a fourfold series, is called a "gauge system" in Weyl's theory. In this analytical scheme, however, zero length is unique, and involves no specification of route. But Eddington, with his idea that it may be possible to introduce non-Maxwellian forces into the schema, further generalises this theory by allowing that zero length may not be unique.

In allowing the generalised idea of measurement of Weyl, and of course, still more in countenancing the suggestion of Eddington, we are abandoning a well-established belief in common sense; and indeed, this is the crux of the matter from the point of view of ordinary life. But this is, of course, not the first time that the theory of relativity has asked us to throw away the beliefs of everyday life. These theoretical developments—and, in fact, the whole of relativity theory—have attained so great a degree of complexity that they have far outstripped the powers of deduction possessed by naïve common sense; and this is so in spite of the fact that they, in common with all other branches of physics, started from ordinary common-sense data. The difficulties, from a common-sense point of view, of the theory of relativity, of which we unfortunately hear so much, are due in great measure to the fact that, owing to the extensive analytical development, the postulates from which it starts have no obvious connection with the physical facts which the theory is designed to correlate. Tensors, for example, involve quantities to which no simple physical significance can at present be attached. But even the concept of energy, which has long since taken its place as a physical idea, must at one stage of history have been a difficult idea to the natural philosopher previously limited to concepts such as force. The concept of *action*, as used in the quantum theory at the present time, is scarcely one which the physicist, left to himself, would readily employ, unless it is regarded as being invariably an angular momentum. The Lagrangian idea of *generalised co-ordinates* in dynamics is another case of the same kind. The concepts employed in relativity are at present remote from physical ideas in exactly the same way, though perhaps to a greater extent.

The Theory of Relativity and Common Sense.

In mathematical theories, not infrequently the logical links between the premises of the problem and the results deduced from them are so many in number that no connection can at first sight be seen between them. In fact, the greater the number of links the more valuable the theory becomes. The purely mathematical background of the theory of relativity consists largely of developments which belong to highly specialised domains; and it is not to be expected that common sense can foresee the results obtainable from specified assumptions which the data of common sense have been found to require. Indeed, we might point out that it is apparent from the mere fact that the tensor theory has been built up into an extensive branch of mathematics (which, of course, happened long before its applications were dreamed of) that the connection between the premises and the results is too complicated to be dealt with without the aid of a specially elaborated technique. It is therefore impolitic to advance common-sense criticisms of the various assumptions as to length which may provisionally be advanced in the theory of relativity with the definite object of effecting further comprehensive correlations of physical facts. For common sense, having provided the jumping-off ground, has a severely restricted part to play in the more technical analysis which the logical development of these assumptions requires; and it is at once the marvel and the allure of the science of our day that mathematics, which is but the child of common sense, has been able, owing to the masterly researches carried on by the pioneers of the nineteenth century, to transform the crude views of her parents into the triumph of modern physics.

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